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ON MULTICOMPONENT FLUID MIXTURES

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1. Introduction

We consider the flow of the mixture of viscous, heat-conducting, compressible fluids consisting of n different components, each of which has its own density, pressure, and molecular weight but all components having the same velocity and temperature. The mathematical model presented here relies on fundamental principles of continuum mechanics and combustion, where the balance laws, the constitutive relations, and other assumptions on the model are based on the following considerations.

(a) We treat the material (*gas*) as a viscous, compressible, heat-conducting fluid, occupying a bounded domain Ω in the physical space $\mathbb{R}^3$, which is a perfect mixture of n species of real monoatomic gases *Fermi gases.*

(b) The motion of the *gas* is an entropy producing (dissipative) process. The transport fluxes satisfy very general constitutive laws taking into account the multicomponent character of the mixture. The reaction rate satisfies a general *Arrhenius-type law* and the transport coefficients depend on the absolute temperature.

(c) The pressure depends on the volume fractions of the individual species; it takes into consideration both the chemical reaction of the individual component as well as radiation effects.

The aim of this work is to establish the global existence and weak asymptotic stability of solutions to the initial boundary value problem with *large* initial data.

This work extends earlier works on the study of the dynamics of mixtures of compressible fluids [37], [38] (see also [15], [16]) since it now treats a physically relevant multidimensional, multicomponent combustion model which complies with the fundamental principles of continuum mechanics

and combustion theory taking into consideration the distinct features of the individual components as they appear in the constitutive laws of the *transport fluxes*, pressure law, internal energy and rate function. The different species are components of different gases with distinct densities, pressures, and molecular weights. The rate function w_k is allowed to be unbounded with respect to both the density and the temperature depending on the concentration of the individual species $k \in S$, while the *transport coefficients*, such as the heat conductivity, the *shear*, and *bulk* viscosity parameters, are functions of the absolute temperature.

Existence results for combustion models as far as the one dimensional case is concerned are presented in a series articles (see Bebernes and Bressan [3]; Bebernes and Eberly [4]; Bressan [5]; Chen [7]; Chen, Hoff, and Trivisa [9], [10], [11]; Ducomet [17], [18]; Ducomet and Zlotnik [21]; Zlotnik [41], and the references therein). Global existence results for weak solutions to a multidimensional combustion model formulated by the Navier-Stokes equations for viscous, compressible, reacting gases are presented by Donatelli and Trivisa [15], [16]. For related articles in the literature, we refer the reader to Feireisl [25], [26] and the references therein. Results on phase transition dynamics for general binary and multicomponent mixtures are presented in the articles by Trivisa [37], [38], [39].

For a survey on the mathematical theory of multicomponent modeling, we refer the reader to Giovangigli [31], while for a survey on the mathematical theory of combustion models, we refer the reader to the manuscripts by Buckmaster [6] and Williams [40].

2. Balance Laws

In this section, we present the fundamental conservation equations governing multicomponent flows which express the conservation of species mass, momentum, and energy.

2.1 Species, momentum, and energy

The balance laws associated with multicomponent reactive flows express the conservation of species mass, momentum, and energy. Species mass conservation can be written in the form

$$(2.1) \qquad \partial_t \rho_k + div(\rho_k \mathbf{u}) + div \mathcal{F}_k = m_k w_k, \quad k \in S$$

where ρ_k is the mass density of the kth species, $\mathbf{u}$ the mass average flow velocity, $\mathcal{F}_k$ is the diffusion flux of the kth species, m_k the molecular weight of the kth species, w_k the reaction rate of the kth component, S is the set of species indices and n the number of species.

By summing the n species conservation equations, we can derive the total mass conservation equation associated with the total density

$$(2.2) \qquad \rho = \sum_{k \in S} \rho_k.$$

In anticipation of the mass constraint

(2.3)
$$\sum_{k \in S} \mathcal{F}_k = 0,$$

between the diffusion fluxes $\mathcal{F}_k, k \in S$, and the mass constraint

(2.4)
$$\sum_{k \in S} m_k w_k = 0,$$

between the reaction rate function $w_k, k \in S$, which will be specified later, we obtain the total mass conservation equation

(2.5)
$$\partial_t \rho + div(\rho \mathbf{u}) = 0.$$

Definition 2.1. *The species mass fractions Y_k, $k \in S$, are defined by*

(2.6)
$$Y_k = \frac{\rho_k}{\rho}, \ k \in S,$$

and satisfy the relation

(2.7)
$$\sum_{k \in S} Y_k = 1.$$

The mass fraction Y_k of the kth species locally represents the mass of the kth species with respect to the total mass mixture. The associated conservation equations now read

$$\partial_t(\rho Y_k) + div(\rho(Y_k \mathbf{u} + \mathcal{F}_k)) = m_k w_k, \ k \in S,$$

which is an alternative representation of the species conservation.

The reaction function w_k determines the character of the combustion and is assumed to satisfy a very general Arrhenius-type law, namely

(2.8)
$$w_k(\rho, \theta, Y_k) = \begin{cases} 0, & 0 \leq \theta \leq \theta_I, \\ c_0 \rho^{m-1} Y_k^m \theta^r e^{-c_1/(\theta - \theta_I)}, & \theta > \theta_I, \end{cases}$$

where c_0, $c_1 > 0$, $r \leq 4$, $m \geq 1$ is the kinetic order, and $\theta_I \geq 0$ is the ignition temperature. As expected, combustion will occur when the temperature rises above the ignition temperature resulting to phase transition which here yields the conversion of some or all of the mass of the reactant (*multicomponent gas*) to product species.

The balance of momentum can be written as

$$\partial_t(\rho \mathbf{u}) + div(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = div\mathbb{S} + \sum_{k \in S} \rho_k \mathbf{b}_k,$$

where ρ is the total mass density, p is the pressure, $\mathbb{S}$ is the *viscous stress tensor*, and $\mathbf{b}_k$ is the external specific force acting on the *kth* species.

The energy conservation equation reads

$$\partial_t(\rho e) + div(\mathbf{u}\rho e) + div\,\mathbf{q} = \mathbb{S} : \nabla u - pdiv\mathbf{u},$$

where $\mathbf{q}$ is the *heat flux*.

In summary, the balance laws governing the multicomponent flow model can be formulated by the Navier Stokes equations in Euler coordinates.

(2.9) $\partial_t \rho + div(\rho \mathbf{u}) = 0$

(2.10) $\partial_t(\rho \mathbf{u}) + div(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = div\mathbb{S} + \sum_{k \in S} \rho_k \mathbf{b}_k$

(2.11) $\partial_t(\rho e) + div(\mathbf{u}\rho e) + div\,\mathbf{q} = \mathbb{S} : \nabla u - p\,div\mathbf{u}$

(2.12) $\partial_t(\rho Y_k) + div(\rho Y_k(\mathbf{u} + \mathcal{F}_k)) = m_k w_k.$

Here $\rho = \rho(t,x), \mathbf{u} = \mathbf{u}(t,x), \theta = \theta(t,x)$ denote the density, the velocity, and the temperature of the mixture, $Y_k = Y_k(x,t)$ and ρ_k are the mass density fraction and the density of the kth component in the fluid at time $t \in \mathbb{R}$ and at the point $x \in \Omega \subset \mathbb{R}^N$, $N = 3$.

The physical properties of the mixture are reflected through various constitutive relations, which specify how the *viscous stress tensor* $\mathbb{S}$, the *heat flux* $\mathbf{q}$, the *pressure* p, and the *internal energy* e are related to the *macroscopic variables* $\rho, \mathbf{u}, \theta, Y_k$.

2.2 TRANSPORT FLUXES

We present now the general form of transport fluxes as given by the kinetic theory of gases. These fluxes are the *viscous stress tensor* $\mathbb{S}$, the *species mass fluxes* $\mathcal{F}_k$, $k \in S$, and the *heat flux* $\mathbf{q}$. They are given in terms of *macroscopic variable gradients* and *transport coefficients* as follows.

  • The viscous stress tensor $\mathbb{S}$ is given by the *Newton's viscosity formula*

(2.13) $\mathbb{S} = \mu \left(\nabla \mathbf{u} + \nabla \mathbf{u}^T - \frac{2}{3}div\mathbf{u}\,\mathbb{I} \right) + \zeta\,div\mathbf{u}\,\mathbb{I},$

where the *shear viscosity* μ and the *bulk viscosity* ζ are supposed to be nonnegative and continuously differentiable functions of the absolute temperature.

  • The *diffusion velocity* $\mathcal{F}_i$ is determined by the law

(2.14) $\mathcal{F}_i = -D\nabla \log(\rho_i \theta).$

  • While the *Fourier's law* for the *heat flux*

$$\mathbf{q}_F = -k(\theta)\nabla\theta$$

is valid for one component systems, for binary and multicomponent mixtures there are other effects besides the thermal conduction which contribute to the heat flux $\mathbf{q}$. When the average velocity of any component i differs from the mass average velocity of the mixture, then $n_i \mathcal{F}_\mathbf{i}$ molecules of type i per unit area per second flow across a surface moving with the mass-average velocity of the gas mixture. The molecules of the individual species will carry across the surface *enthalpy*. The total enthalpy (of all species) per unit area per second flowing relative to the mass-average motion of the mixture is

$$\frac{9\gamma - 5}{4} \sum_{i=1}^{N} h_i \rho Y_i \mathcal{F}_i\,.$$

This term constitutes an additional contribution to $\mathbf{q}$ in binary and multicomponent systems. Taking the above discussion into consideration, the heat flux $\mathbf{q} = \mathbf{q}(\rho, \rho_k, \theta, \nabla\theta)$ is given in the case of a mixture of *Fermi gases* $\gamma = \frac{5}{3}$ by a general law

$$(2.15) \qquad \mathbf{q} = -\kappa(\theta)\nabla\theta - \frac{5}{2}LD\rho_k\theta\nabla\log(\rho_k\theta),$$

with D the species diffusion coefficient and L a constant that will be specified in the sequel.

2.3 THE STATE EQUATION OF THE PRESSURE

Modeling multicomponent flows requires introducing molar quantities. The mole fraction of the kth species X_k is defined by

$$(2.16) \qquad X_k = \frac{\bar{m}}{m_k}Y_k, \ k \in S,$$

which gives us the number of moles of the kth species with respect to the number of moles in the mixture. Other molar quantities are the species partial pressures. Here, and in what follows, we consider a *Dalton-type law* of the form

$$(2.17) \qquad p = \sum_{k=1}^{n} \frac{2}{3}\frac{\rho_k e}{m_k} + \frac{a}{3}\theta^4, \quad k \in S.$$

If we denote by m_j the molecular weight of the heaviest component in the mixture, that is $m_j >> m_k$ for all k, then

$$(2.18) \qquad p = \frac{2}{3}\frac{\rho e}{m_j} + \frac{2}{3}\left[\sum_{j\neq k=1}^{n} \frac{\rho_k}{m_k}\left(1 - \frac{m_k}{m_j}\right)e\right] + \frac{a}{3}\theta^4,$$

or equivalently,

$$(2.19) \qquad p = \frac{2}{3}\frac{\rho e}{m_j} + \sum_{j\neq k=1}^{n} \frac{2}{3}L_k\rho_k e + \frac{a}{3}\theta^4,$$

where

$$L_k = \frac{1}{m_k}\left(1 - \frac{m_k}{m_j}\right).$$

Taking the above discussion into consideration, the pressure of the mixture is given as the sum of three parts

$$(2.20) \qquad p = p_F + p_k + p_R,$$

where

$$(2.21) \qquad \begin{cases} p_F = \frac{2}{3}\rho e_F, \quad \text{or} \quad p_F = \theta^{5/2}P\left[\frac{\rho}{\theta^{3/2}}\right], \\ p_k = \sum_k L_k\theta^{5/2}P_k\left[\frac{\rho_k}{\theta^{5/2}}\right], \\ p_R = \frac{a}{3}\theta^4, \end{cases}$$

where the term p_R accounts for *radiation effects* and p_k accounts for the presence of the individual components. In the above relations, P and P_k are given functions which satisfy the following assumptions

$$(2.22) \quad \begin{cases} P \in C^1[0, \infty), \ P(0) = 0, P'(z) \geq 0, \quad \text{for all} \quad z > 0, \\ 0 < \frac{5}{3}P(z) - P'(z)z \leq cz^{\frac{1}{3}} \quad \text{for all} \quad z > 0, \\ \lim_{z \to \infty} \frac{P(z)}{z^{\frac{5}{3}}} \equiv P_\infty. \end{cases}$$

2.4   SPECIFIC INTERNAL ENERGY

In accordance with the above discussion and standard thermodynamic principles, the *specific internal energy* is now given as

$$(2.23) \qquad\qquad\qquad e = e_F + e_k + e_R$$

with

$$(2.24) \qquad e_F = \frac{3}{2}\frac{p_F(\rho, \theta)}{\rho}, \quad e_k = \frac{3}{2}\frac{p_k(\rho_k, \theta)}{\rho}, \quad e_R = \frac{a}{\rho}\theta^4.$$

2.5   THERMODYNAMICS - ENTROPY PRODUCTION

The *internal energy* is related with the *specific entropy s* through a rather general *thermodynamic relation*

$$(2.25) \qquad \theta\mathbf{D}s = \mathbf{D}e + p\mathbf{D}\left(\frac{1}{\rho}\right) - g(\rho, \rho_s, \theta, Y_k)\mathbf{D}Y_k,$$

where $\mathbf{D}$ denotes the total differential, and g a function, which depends on the specific *physical context.*

The *specific entropy s* is given by

$$(2.26) \qquad\qquad\qquad s = s_F + s_k + s_R$$

where

$$(2.27) \quad s_F = S_F\left(\frac{\rho}{\theta^{3/2}}\right), \quad s_k = \sum_k S_k\left(\frac{\rho_k}{\theta^{3/2}}\right), \quad s_R = \frac{4a}{3}\frac{\theta^3}{\rho}$$

with

$$(2.28) \qquad\qquad S_F'(z) = -\frac{3}{2}\frac{\frac{5}{3}P(z) - P'(z)z}{z^2}.$$

If the motion is smooth, starting from the *energy balance* equation and in accordance with standard thermodynamic principles, we derive the *entropy* equation, which now reads,

$$\partial_t(\rho s) + div(\rho s\mathbf{u}) + div\left(-\frac{k(\theta)}{\theta}\nabla\theta - (\rho_k s_k)KD\nabla\log(\rho_k\theta)\right) = \mathbf{r}.$$
$$(2.29)$$

Here $\mathbf{r}$ denotes the *entropy production*, which is expressed by

$$(2.30) \quad \mathbf{r} = \frac{1}{\theta}\left(\mathbb{S} : \nabla u + \frac{k(\theta)|\nabla\theta|^2}{\theta} + K\rho_k\theta D|\nabla\log(\rho_k\theta)|^2\right).$$

Replacing the balance of energy equation by the balance of entropy equation, the system (2.9)–(2.12) now reads

$$
\begin{cases}
\partial_t \rho + div(\rho \mathbf{u}) = 0 \\
\partial_t(\rho \mathbf{u}) + div(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = div\mathbb{S} + \sum_{k \in S} \rho_k \mathbf{b}_k \\
\partial_t(\rho s) + div(\rho s \mathbf{u}) + div\left(-\frac{k(\theta)}{\theta}\nabla\theta - (\rho_k s_k)KD\nabla\log(\rho_k\theta)\right) = \mathbf{r}. \\
\partial_t(\rho Y_k) + div(\rho Y_k(\mathbf{u} + \mathcal{F}_k)) = m_k w_k.
\end{cases}
$$

In the case of a general nonsmooth motion, now and in the spirit of the second law of thermodynamics, we can only assert that

$$
\partial_t[\rho s] + div([\rho s]\mathbf{u}) + div\left(-\frac{k(\theta)}{\theta}\nabla\theta - [\rho_k s_k]KD\nabla\log(\rho_k\theta)\right)
$$

(2.31)
$$
\geq \frac{1}{\theta}\left(\mathbb{S} : \nabla u + \frac{k(\theta)|\nabla\theta|^2}{\theta} + K\rho_k\theta D|\nabla\log(\rho_k\theta)|^2\right).
$$

We assume that the mixture occupies a bounded domain $\Omega \subset \mathbb{R}^N$, $N = 3$ of class $C^{2+\nu}$, $\nu > 0$, and the whole physical system is both mechanically and thermally isolated. The following boundary conditions hold

(2.32)
$$
\mathbf{u}|_{\partial\Omega} = 0, \quad \mathbf{q} \cdot \mathbf{n}|_{\partial\Omega} = 0, \quad \mathcal{F} \cdot \mathbf{n}|_{\partial\Omega} = 0,
$$

where $\mathbf{n}$ denotes the outer normal vector to $\partial\Omega$.

In accordance with the above discussion, we infer that the *total energy*

(2.33)
$$
E = \frac{1}{2}\rho|\mathbf{u}|^2 + \rho e + \rho Y_k
$$

is constant of motion, specifically,

(2.34)
$$
\frac{d}{dt}\int_\Omega E(t)dx = 0.
$$

We consider the following initial conditions:

(2.35)
$$
\begin{cases}
\rho(0, \cdot) = \rho_0, \\
(\rho\,\mathbf{u})(0, \cdot) = \mathbf{m}_0, \\
(\rho\,\theta)(0, \cdot) = \rho_0\theta_0, \\
(\rho\,Y_k)(0, \cdot) = \rho_0 Y_{k0},
\end{cases}
$$

together with the compatibility condition:

(2.36)
$$
\mathbf{m}_0 = 0 \quad \text{on the set} \quad \{x \in \Omega|\ \rho_0(x) = 0\}.
$$

The objective of this work is to establish the global existence of weak solutions to this initial boundary value problem with large initial data.

3. Variational Formulation

Our goal is to solve the initial-boundary value problem presented above with *large* initial data. We rely on the concept of *variational solution*, which in spirit seems to be much closer to the underlying physical principles, known as *balance laws*, stated in the form of integral identities without any assumption of smoothness of the macroscopic variables.

In the framework of weak solutions, and in accordance with the second law of thermodynamics, it is common to replace the (formally derived) *classical* (thermal, energy or entropy) *equality* by an *inequality*. The underlying idea is that part of the kinetic energy may disappear in the form of a positive measure and become part of the domain. For further remarks on the topic, we refer the reader to Feireisl [25] for relevant discussion.

Motivated by this discussion, we introduce now the notion of a *variational solution* to the initial boundary value problem (2.9)-(2.12) together with (2.32) and (2.35).

4. The Main Result

Our goal is to solve of the initial-boundary value problem presented above with *large* initial data.

4.1 Variational solutions

Definition 4.1. *We say that* $(\rho, \mathbf{u}, \theta, X_k)$ *with*

$$\rho \in L^\infty(0, T; L^\gamma(\Omega)), \ \mathbf{u} \in L^2(0, T; W_0^{1,2}(\Omega)), \ \theta, \in L^2(0, T; W^{1,2}(\Omega)),$$

$$X_k \in L^\infty((0, T) \times \Omega) \cap L^2(0, T; W^{1,2}(\Omega))$$

is a variational solution of (2.9)-(2.12) *if*

      (a) *The density* ρ *is a nonnegative function on* $(0, T) \times \Omega$,

$$\log \theta \in L^2(0, T; W^{1,2}(\Omega)),$$

    and

$$0 \le X_k(t, x) \le 1 \quad \text{for a.a.} \quad (t, x) \in (0, T) \times \Omega.$$

The continuity equation (2.9) *is satisfied in* $\mathcal{D}'((0, T) \times \mathbb{R}^3)$ *provided* $\rho, \mathbf{u}$ *were extended to be zero outside* Ω. *In addition, we require that the density* ρ *(and the velocity* $\mathbf{u}$) *are renormalized solutions of the continuity equation, in the sense that they satisfy the* **renormalized continuity equation**

(4.1) $\qquad \partial_t b(\rho) + div\,(b(\rho)\mathbf{u}) + \big(b'(\rho)\rho - b(\rho)\big)\,div\mathbf{u} = 0,$

in $\mathcal{D}'((0, T) \times \Omega)$, *for any smooth* $b \in C^1[0, \infty)$ *such that* $b'(z) = 0$ *for all* $z \ge z_b$.

      (b) *The balance of momentum* (2.10) *is satisfied in* $\mathcal{D}'((0, T) \times \Omega)$.

(c) *The specific entropy s satisfies the integral relation*

$$\int_0^T \int_\Omega \rho s \partial_t \phi + \rho s \mathbf{u} \cdot \nabla \phi + \nabla \phi \left[-\frac{k(\theta)}{\theta} \nabla \theta - (\rho_k s_k) LD \nabla \log(\rho_k \theta) \right] + \mathbf{r} \phi \, dxdt \leq 0.$$

with $s = s_F + s_k$ *given by (2.27) and* $\mathbf{r}$ *determined by (2.29).*

(d) *The species mass fractions* X_k *satisfies the integral relation*

(4.2)
$$\int_0^T \int_\Omega \rho_k \phi_t + \rho_k \mathbf{u} \nabla \mathbf{u} - \rho_k D \nabla \log(\rho_k \theta) \, dxdt = 0$$

for any test function $\phi \in \mathcal{D}((0,T) \times \bar{\Omega})$.

(e) *As there is no flux of energy through the kinematic boundary, the total energy* $E(t)$ *given by (2.33) is constant of motion, that is*

(4.3)
$$\int_0^T E(t) \, \partial_t \psi \, dt = 0, \quad \text{for any} \quad \psi \in \mathcal{D}((0,T)).$$

4.2 STRUCTURAL CONDITIONS ON THE VISCOSITY COEFFICIENTS

1. The viscosity parameters μ and ζ are nonnegative, continuously differentiable functions of the absolute temperature satisfying

(4.4)
$$\begin{cases} 0 < \underline{\mu}(1 + \theta^\beta) \leq \mu(\theta) \leq \bar{\mu}(1 + \theta^{\beta-1}), \\ \zeta \geq 0, \quad \underline{\zeta}\theta^\beta - 1 \leq \zeta(\theta), \quad |\zeta'(\theta)| \leq \bar{\zeta}(1 + \theta^{\beta-1}) \leq \bar{\zeta}(1 + \theta^\alpha) \end{cases}$$

for all $\theta > 0$. In the above relation, $\underline{\mu}, \underline{\zeta}, \bar{\mu}, \bar{\zeta}$ are positive constants.

2. The heat conductivity can be assumed to satisfy

(4.5)
$$k(\rho, \theta) = k_p(\theta) + k(\rho, \theta),$$

with

(4.6)
$$\begin{cases} 0 < \underline{k} \leq k_p(\theta) \leq \bar{k}(1 + \theta^\beta), \\ \underline{k}\theta^\alpha \leq k_R(\rho, \theta) \leq \bar{k}(1 + \theta^\alpha) \end{cases}$$

for all $\rho, \theta > 0$, and for positive constants $\underline{k}, \bar{k}$, while the exponents α and β will satisfy some technical conditions [39].

5. THE MAIN RESULT

We are now ready to state our main result.

Theorem 5.1. *Let* $\Omega \subset \mathbb{R}^3$ *be a bounded domain with a boundary* $\partial\Omega \in C^{2+\nu}, \nu > 0$. *Suppose that the pressure* p *is determined by the equation of state (2.19). In addition, let the viscous stress tensor* $\mathbb{S}$ *be given by (2.13), with* μ *and* ζ *continuous differentiable functions of* θ *satisfying (4.4). Similarly, let the heat flux* $\mathbf{q}$ *be given by (2.15) with the heat conductivity* k *satisfying (4.5) and (4.6). Then for any initial data*

$$\rho_0, \mathbf{u}_0, \theta_0, X_{k_0} \in L^\infty(\Omega; \mathbb{R}^3),$$

such that

(5.1) $\rho_0(x) \geq 0, \quad 0 < \underline{\theta} \leq \theta_0(x) \leq \bar{\theta}, \quad 0 \leq X_s(x) \leq 1 \quad$ *for a.a.* $\quad x \in \Omega,$

the initial-boundary value problem (2.9)-(2.12) *has for* $\gamma > \frac{5}{3}$ *at least one variational solution on* $(0,T) \times \Omega$ *in the sense of Definition 4.1 satisfying the initial conditions*

(5.2)
$$
\begin{cases}
\rho(t) \to \rho_0 \quad in \quad L^1(\Omega), \\
\mathbf{m}(t) \to \mathbf{m}_0 \quad weakly \ in \quad [L^1(\Omega)]^3, \\
(\rho X_k)(t) \to \rho_0 X_{k_0} \quad weakly \ in \quad L^1(\Omega), \\
\qquad\qquad as \quad t \to 0+ .
\end{cases}
$$

(5.3) $ess \lim\limits_{t \to 0+} \int_\Omega (\rho s)(t)\phi \, dx \geq \int_\Omega \rho_0 s_0 \phi dx \quad for \ any \quad \phi \geq 0, \phi \in \mathcal{D}(\Omega).$

5.1 General strategy

Our strategy involves:

- Constructing a series of approximating problems by regularizing the equations (and the equations of state for the pressure, internal energy and entropy) and considering appropriate initial and boundary conditions. The regularization appears in terms of additional ε and δ in the equations. The form of this regularization depends on the choice of constitutive relations.
- Deriving all the possible *a priori* bounds (energy and entropy estimates) for the sequence of approximate solutions.
- Deriving improved thermal and pressure estimates by extending the *multipliers technique* of Feireisl [25], P.-L. Lions [35]. Having established the weak continuity of the viscous stress tensor, we let $\varepsilon \to 0$ and $\delta \to 0$ in order to get rid of the artificial viscosity and to recover the original system. This process becomes possible because of the strong convergence of the density sequence.

Proof. For the proof we refer the reader to Trivisa [39]. $\square$

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