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THE ONSET OF DOUBLE-DIFFUSIVE CONVECTION IN A HORIZONTAL LAYER OF A POROUS MEDIUM IN THE PRESENCE OF VIBRATION

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The present work investigates the effect of vertical harmonic vibration on the onset of convection in an infinite horizontal layer of a binary fluid mixture saturating a porous medium. We assume that the fluid component of the mixture is viscous and incompressible, the porous medium is homogeneous and isotropic, and the boundaries are rigid and impermeable, with slip allowed. A constant temperature and concentration distribution is specified on the boundaries.

The mathematical model is described by equations of filtrational convection (see [4], [2]) in the Darcy-Oberbeck-Boussinesq approximation. In the coordinate system inflexibly fixed to the oscillating horizontal layer, with the z-axis directed vertically upward, the governing equations have the following form:

$$\frac{1}{\varphi}\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\nabla p - \frac{\nu}{K}\mathbf{v} + g(t)\left(\beta T - \beta_C C\right)\mathbf{k}, \tag{1}$$

$$\varkappa\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \chi\nabla^2 T, \tag{2}$$

$$\varphi\frac{\partial C}{\partial t} + \mathbf{v}\cdot\nabla C = D_m\nabla^2 C, \tag{3}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{4}$$

where $\mathbf{v}$ is the relative filtration velocity of the fluid mixture, p is the convective pressure, T is the temperature of the fluid mixture and solid phase, C is the concentration of the heavier component of the mixture, φ is the porosity of the medium, ρ is the density of the fluid mixture, ν is the kinematic viscosity of the fluid mixture, K is the intrinsic permeability of the porous medium, β is the coefficient of thermal expansion of the fluid mixture, β_C is the coefficient of concentration expansion of the fluid mixture, $\mathbf{k}$ is the unit vector directed upward, $\varkappa$ is the heat capacity ratio, χ is the thermal diffusivity of the porous medium, and D_m is the mass diffusivity of the porous medium. The gravitational field $g(t)$ consists of two parts:

$g(t) = g_0 + g_e(t)$, with g_0 being the steady acceleration due to the static gravity, and $g_e(t) = \frac{A}{\varphi}\,\Omega^2 f''(\Omega t)$ the vibrational acceleration. Here, A is the amplitude, Ω is the frequency of vibration, and $f(\Omega t)$ is a 2π-periodic function with zero 2π-average.

The system $(1)-(4)$ has a solution corresponding to the quasi-equilibrium basic state

$$\mathbf{v}^0 = 0, \quad T^0 = T_1 - \frac{1}{h}(T_1 - T_2)z, \quad C^0 = C_1 - \frac{1}{h}(C_1 - C_2)z,$$

$$(5)\ \ p^0 = \rho g(t)\left[(\beta T_1 - \beta_C C_1)z - \frac{1}{2h}\big(\beta(T_1 - T_2) - \beta_C(C_1 - C_2)\big)z^2\right],$$

where T_1, C_1 and T_2, C_2 are the assigned values of temperature and concentration on the lower ($z = 0$) and upper ($z = h$) boundaries, respectively. The linear stability analysis for this solution is performed by using Floquet theory (see [6]). The system of equations for normal disturbances includes the following nondimensional parameters: the ordinary (filtration) Grashoff number Gr and its concentration analog $\mathrm{Gr_c}$, the Prandtl number Pr, the Schmidt number Sc, and the amplitude η and frequency ω of vibration.

After separating the space variables, we obtain a system of ODE's with periodic coefficients. Representing its solution in the form of a Fourier series (in time), we derive an infinite tri-diagonal system of linear algebraic equations for the Fourier coefficients. Employment of the continued fractions method to solve this tri-diagonal system allows us to derive the dispersion equation for the Floquet exponent in explicit form (see [3], [7], [8], [5]). This equation is investigated analytically and numerically for different values of vibration parameters, as well as the thermal Rayleigh number, $\mathrm{Ra} = \mathrm{Gr}\cdot\mathrm{Pr}$, and concentration Rayleigh number, $\mathrm{Rs} = \mathrm{Gr_c}\cdot\mathrm{Sc}$. The marginal stability curves of the thermal Rayleigh number versus horizontal wave number α are constructed for the synchronous and subharmonic resonant modes over different values of the concentration Rayleigh number and the amplitude and frequency of vibration (see Figure 1). Regions inside these curves indicate the resonant instability of the quasi-equilibrium basic state (5).

In the case of high frequency vibration, the method of averaging is applied to investigate the stability of the basic state (5) (see [1], [9], [10]). The dispersion equation for the Floquet exponent is reduced to a cubic equation. Asymptotic formulas for the marginal stability curves of the thermal Rayleigh number versus horizontal wave number are derived for the monotonic disturbances (synchronous and subharmonic modes), and oscillatory disturbances. It is shown that in the case of high frequency vibration, there exist closed marginal curves bounding the instability regions (see Figure 2).

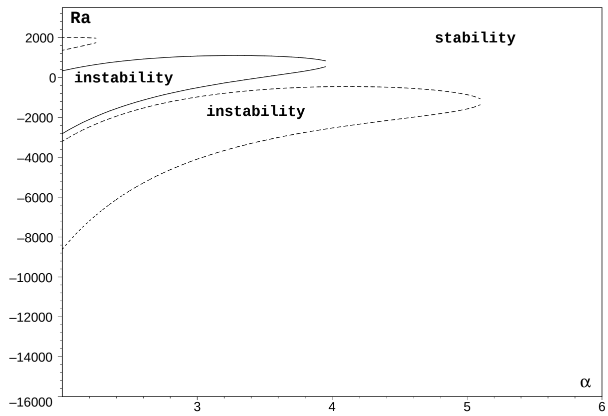

FIGURE 1. Marginal curves of Ra versus α for synchronous (solid line) and subharmonic (dashed line) modes, obtained by the method of continued fractions; $\omega = 30,\ \eta = 3,\ \mathrm{Rs} = 3000,\ \mathrm{Le} = 1$.

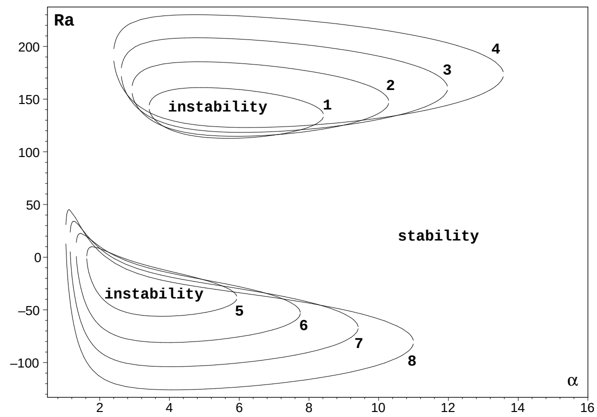

FIGURE 2. Marginal curves of Ra versus α for large ω, obtained by the method of averaging for the case of monotonic instability; $\eta/\omega = 0.3,\ \mathrm{Le} = 3$. Curves 1– 8 correspond to 1) $\mathrm{Rs} = 163$, 2) $\mathrm{Rs} = 183$, 3) $\mathrm{Rs} = 203$, 4) $\mathrm{Rs} = 223$, 5) $\mathrm{Rs} = -59$, 6) $\mathrm{Rs} = -79$, 7) $\mathrm{Rs} = -99$, 8) $\mathrm{Rs} = -119$.

Analytical and numerical investigation of the problem shows that vertical vibration can either stabilize or destabilize the system (by means of delaying or inducing convection), depending on the frequency and amplitude of vibration.

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