

TENTATIVE LIST OF 620-22 TOPICS (AND SOME 520-21-22 TOPICS) TO BE REVIEWED FOR 1994 WRITTEN GENERAL EXAM IN ANALYSIS:

PRIMARY TOPICS FOR WHICH YOU SHOULD KNOW DEFINITIONS, STATEMENTS OF THEOREMS, EASIER SELF-CONTAINED PROOFS, AND EXAMPLES:

1. Continuity and differentiability of real functions. Sets of continuity. Continuous nowhere differentiable functions.
2. Monotone functions and functions of bounded variation.
3. Cardinality, countable sets, uncountable sets, sets of cardinality c , Bernstein's Theorem (not the proof).
4. Riemann integral, characterization (bounded, discontinuity set of measure zero) of Riemann integrability of a function on an interval.
5. Sigma-algebras, minimal sigma-algebra containing a given collection of sets, the Borel sets. Finite countably additive non-negative measure μ on a sigma-algebra.
6. Lebesgue outer measure λ^0 and Lebesgue measure λ on an interval and on the reals. Non-measurable sets (Vitali and Bernstein). Approximation to the measure of a set M with a closed set inside M and an open set containing M .
7. Perfect sets and Cantor sets (of measure zero and of positive measure).
8. Nowhere dense sets, first and second category sets, residual sets, the Baire Category Theorem, first category sets of full measure.
9. Measurable functions, basic theorems ($f+g$ measurable etc.), Lusin's Theorem.
10. Sequences of measurable functions. Uniform, pointwise, almost everywhere, and L^p -convergence (primarily L^1 -, L^2 -, and L^∞ -convergence). Convergence in measure. Egoroff's Theorem.
11. Lebesgue integral (with respect to Lebesgue measure and with respect to a finite non-negative measure). Dominated Convergence Theorem and Monotone Convergence Theorem (statements and related examples).
12. C^1 functions, absolutely continuous functions ($\varepsilon - \delta$ partition definition and equivalent Lebesgue integral definition), Lip^1 functions, and CBV functions (definitions, relations to each other, and examples, including Cantor function).
13. L^p spaces (primarily L^1 , L^2 and L^∞), with respect to Lebesgue measure on $[0,1]$ and with respect to an arbitrary finite non-negative measure. Hölder's inequality in L^p and Schwarz's inequality in L^2 . (also l^p).
14. Product measures and Fubini's Theorem (Statements and examples).
15. Vitali Covering Theorem, Lebesgue Density Theorem, and Lebesgue Differentiability Theorem (not the proofs).