TOPOLOGY PRELIMINARY EXAMINATION
May 18 1996.

I. Do three of the first four problems.

II. Then do five other problems (this may include whichever of the first four problems that you have not done for part I).

Please provide a cover sheet which lists the eight problems to which you have provided solutions. Please have your solutions to different problems on separate sheets of paper. Place your name on all the sheets you hand in.

Problem 1. Let X be a compact Hausdorff space and let G be a collection of closed subsets of X such that if g is in G, then every point of g is a limit point of $X - g$. Show that if $X = \cup G$, then G is not countable.

Problem 2. Let X be metric space with metric d and suppose that H and K are compact subsets of X. Show that there exist points p and q in H and K respectively so that if x and y are any other pair of points from H and K respectively then $d(p, q) \leq d(x, y)$.

Problem 3. Show that if X is regular and Y is regular then $X \times Y$ is regular.

Problem 4. Suppose that X is a compact Hausdorff space and G is a monotonic collection of closed sets in X. (G is *monotonic* means: if H and K are two elements of G then one is a subset of the other.) Show:

a. There is a point in X common to all the elements of G.

b. If each element of G is connected then so is the common part of the elements of G.

Problem 5. Show that paracompactness implies regular. (Suppose that X is a space and G is a collection of subsets of X. Then G is said to be *locally finite* iff for each point x of X there is an open set U containing X so that U intersects only finitely many elements of G. The topological space X is said to be *paracompact* iff it is Hausdorff and every open cover of X has a locally finite refinement that covers X.)

Problem 6. Let X be a metric space and suppose that E is a compact metric space that is mapped onto X by a local homeomorphism p. Then show that E is a covering space of X.

Problem 7. Show that paths can be lifted to covering spaces.

Problem 8. Show that a strong deformation retract of the plane is simply connected.

Problem 9. Let X be an uncountable well-ordered set and suppose that X has the order topology. Show that there is a subspace of X which is limit compact but not compact. (A set is said to be *limit compact* iff every infinite subset of the set has a limit point in the set.)

Problem 10. Let X be a compact connected Hausdorff space and let p, q, and r be three distinct points of X. Show that there is a subcontinuum M of X containing p, q, and r so that if H is a proper subcontinuum of M then H misses one of p, q, or r. (Hint use Zorn's lemma).

Problem 11. Show that components of open sets of a locally connected Hausdorff space are open.

Problem 12. Give an example of a regular space which is not normal.